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# Fuzzy decision-making on reliability of series system: Fuzzy geometric programming approach

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Abstract. In this paper, we have presented different type of fuzzy geometric programming based on fuzzy decision-making through max-min and max-additive operators. These operators are applied in the series system reliability model in fuzzy environment. In the problem we have considered reliability of series system with limited system cost as a constraint function. Numerical examples are given to illustrate fuzzy decision-making based fuzzy geometric programming on series system reliability model.

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## 1. Introduction

In real life, the data can not be recorded or collected precisely due to human errors or some unexpected situations. So one may consider ambiguous situations like vague parameters, non-exact objective and constraint functions in the problem and it may be classified as a non-stochastic imprecise model. In the earliest stage of system design the reliability decision is usually made on precise data but in real life problem, available data is incomplete and imprecise in nature. Here fuzzy set theory may provide a method to describe or formulate this imprecise model. Reliability is one of the vital attributes of performance in arriving at the optimal design of a system because it directly and significantly influences the system's performance. In practical, the problem of series system reliability may be formed as a typical non-linear programming problem with non-linear cost-functions in fuzzy environment. Some researchers applied the fuzzy set theory to reliability analysis. Park [18] used fuzzy in the reliability apportionment problem for a two-component series system subject to a single constraint and solved it by Fuzzy Non-linear Programming (FNLP) technique. Huang [13] presented a fuzzy multi-objective optimization decision-making method of reliability of series system. Mahapatra and Roy [15] introduced fuzzy multi-objective mathematical programming technique based on generalized fuzzy set and they applied it in series system reliability optimization model. Ruan and Sun [19] presented an exact method for cost minimization problems in series reliability systems with multiple component choices. Nahas and Nourelfath [17] presented an application of ant system in a reliability optimization problem to maximize the system reliability for a series system subject to the system budget. Sung and Cho [20] used branch-and-bound solution algorithm on a series system reliability optimization problem for maximize the system reliability with limited system budget. Ahmadizar and Soltanpanah [2] presented reliability optimization of a series system with multiple-choice and budget constraints using ant colony approach. Many researchers have presented different situations and solutions techniques on series system reliability optimization model [4, 12, 14, 25] in different environments. The non-linear optimization problems have been solved by various non-linear optimization techniques. Geometric Programming (GP) [8, 24] is an effective method among those to solve a particular type of non-linear programming problem. Cao [6] discussed Fuzzy Geometric Programming (FGP) with zero degree of difficulty. GP method is rare used to solve the reliability optimization problem. Federowiez and Mazumder [9] first used GP on reliability optimization problem. Govil [10] used GP for a 3-stage series reliability system. Govil [11] also applied GP method on an optimal maintainability problem of a series system with cost constraint. In 1987, Cao [5] first introduce FGP. There is a good book dealing with FGP by Cao [7]. Mahapatra and Roy [16] used FGP with cost constraint to find optimal reliability for a series system using linear and non-linear membership functions of system cost and reliability. Fuzzy reliability optimization model through FGP is very rear in literature. An optimal reliability design is one in which all possible means available to a designer have been explored to enhance the reliability of the system with limited available system cost. In this paper we have considered reliability problem of series system model and used FGP to find optimal system reliability which gives the choice to Decision-Maker (DM) to get the optimal reliability subject to the limited system cost. The rest of the paper is organized as follow: In section 2, the series system reliability model is presented in crisp and fuzzy environment. Section 3, associate preliminary mathematics has been presented. In section 4, FGP is presented based on fuzzy decision-making concepts with max-min and max-additive operators. In section 5, application of FGP technique on series system reliability optimization problem is presented. Numerical example is given to demonstrate FGP in series system in section 6. Some conclusions are made in section 7.

## 2. Formulation of Series System Reliability Model

Let us consider the reliability problem of a series system consists of $n$ components. Each component has reliability $R_i$ for the $i$th components for $i = 1, 2, \cdots, n$. The system reliability is the product of reliability of each component. The system reliability can be written as $R_s(R_1, R_2, \cdots, R_n) = R_1 R_2 \cdots R_n$. According to Aggarwal et al. [1] the cost of reliability is monotonically increasing function of reliability. Based on Tillman et al. [21, 22] the $i$th components reliability cost is $C_i R_i^{a_i}$ where $a_i < 1$ and $C_i$ are shape parameters of $i$th component for $i = 1, 2, \cdots, n$. Hence the system cost is given by $C_s(R_1, R_2, \cdots, R_n) = \sum_{i=1}^{\infty} C_i R_i^{a_i}$. Let $C_s$ gives the maximum permissible system cost. So the cost constraint function can be written as follows $C_s(R_1, R_2, \cdots, R_n) \equiv \sum_{i=1}^{\infty} C_i R_i^{a_i} \leq C_s$.

2.1. **Crisp form of Series System Reliability Optimization Model.** To solve by GP technique the reliability problem of series system should be in minimization form. Hence the suitable form of optimization model of series system reliability problem to solve by GP technique is taken as

$$Min\, R_s(R_1, R_2, \cdots, R_n) = R_1^{-1} R_2^{-1} \cdots R_n^{-1} \tag{2.1}$$

subject to $\sum_{i=1}^{n} C_i R_i^{a_i} \leq C_s$, $0 < R_i \leq 1$ for $i = 1, 2, \cdots, n$.

Here objective and constraint functions are polynomial form. So we can use GP to solve this problem.

2.2. **Series System Reliability Optimization Model in Fuzzy Environment.** The necessary features of the cost vs. maintainability function are equivalent to the cost vs. reliability function as given by Aggarwal et.al. [1]. It is very complicated decision making process to determining the reliability components in fuzzy objective as well as constraint goal. It involves many uncertain factors and becomes a non-stochastic vague decision making process. Therefore the reliability allocation model (2.1) can be represented by fuzzy non-linear programming to make the model more flexible and adoptable to the human decision process.

In fuzzy environment the reliability optimization problem (2.1) becomes

$$\tilde{Min}\, R_s(R_1, R_2, \cdots, R_n) = R_1^{-1} R_2^{-1} \cdots R_n^{-1} \tag{2.2}$$

subject to $\sum_{i=1}^{n} C_i R_i^{a_i} \lesssim C_s$, $0 < R_i \leq 1$ for $i = 1, 2, \cdots, n$.

## 3. Prerequisites Mathematics

Fuzzy set was introduced by Zadeh [23] as a mathematical way of representing impreciseness or vagueness.

**Definition 3.1.** A fuzzy set $\tilde{A}$ in a universe of discourse $X$ is defined as the set of pairs

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid x \in X\},$$

where $\mu_{\tilde{A}} : X \to [0, 1]$ is a mapping called the *membership function* of the fuzzy set $\tilde{A}$ and $\mu_{\tilde{A}}(x)$ is called the *membership value* or *degree of membership* of $x \in X$ in the fuzzy set $\tilde{A}$.

3.1. **Idea of Fuzzy Decision Making.** In this real world most of the decision making problems takes place in a fuzzy environment. The objective goal, constraints and the consequences of possible actions are not known precisely. Under this observation, Bellman et al. [3] introduced three basic concepts. They are fuzzy objective goal, fuzzy constraint and fuzzy decision based on fuzzy goal and constraint. We introduce the conceptual framework for decision making in a fuzzy environment.

Let $X$ be a given set of possible alternatives which contains the solution of a decision making problem in fuzzy environment. The problem based on fuzzy decision making may be considered as follows.

Optimize fuzzy goal $\tilde{G}$,
Subject to fuzzy constraint $\tilde{C}$ for $j = 1, 2, \cdots, m$.

A fuzzy goal $\tilde{G} = \{(x, \mu_{\tilde{G}}(x)) \mid x \in X\}$ and a fuzzy constraint

$$\tilde{C} = \{(x, \mu_{\tilde{C}}(x)) \mid x \in X\}$$

is a fuzzy set on $X$ characterized by its membership function $\mu_{\tilde{G}} : X \to [0, 1]$ and $\mu_{\tilde{C}} : X \to [0, 1]$, respectively. Both the fuzzy goal and fuzzy constraint are desired to be satisfied simultaneously. So Bellman et al. [3] defined fuzzy decision through fuzzy goal and fuzzy constraint.

**Definition 3.2.** *Fuzzy decision* based on min-operator $\tilde{D}_m = \left\{ \left(x, \mu_{\tilde{D}_m}(x)\right) \mid x \in X \right\}$ is a fuzzy set defined as $\tilde{D}_m = \tilde{G} \cap \tilde{C}$. It is characterized by

$$\mu_{\tilde{D}_m}(x) = \min \{\mu_{\tilde{G}}(x), \mu_{\tilde{C}}(x)\}$$

for all $x \in X$. So, maximizing decision is defined as

$$Max\, \mu_{\tilde{D}}(x^*) = \max_{x \in X} \{\min \{\mu_{\tilde{G}}(x), \mu_{\tilde{C}}(x)\}\}.$$

There is another aggregation pattern, which is additive fuzzy decision. Fuzzy decision based on additive operator is a fuzzy set $\tilde{D}_a = \left\{ \left(x, \mu_{\tilde{D}_a}(x)\right) \mid x \in X \right\}$ where $\mu_{\tilde{D}_a}(x) = \mu_{\tilde{G}}(x) + \mu_{\tilde{C}}(x)$ for all $x \in X$.

## 4. Geometric Programming

Let the general form of GP problem be considered as follows.
Find $x = (x_1, x_2, \cdots, x_n)^T$ of the following problem:

$$\text{Minimize } f_0(x) = \sum_{k=1}^{T_0} \sigma_{0k} c_{0k} \prod_{r=1}^{n} x_r^{\alpha_0 kr} \tag{4.1}$$

Subject to

$$\begin{cases} f_j(x) = \sum\limits_{k=1}^{T_j} \sigma_{jk} c_{jk} \prod\limits_{r=1}^{n} x_r^{\alpha_j kr} \le \zeta_j f_j^0 & \text{for } j = 1, 2, \cdots, m, \\ x_r > 0 & \text{for } r = 1, 2, \cdots, n \end{cases}$$

where $c_{rk}$, $f_j^0 (> 0)$, $\operatorname{sign} \sigma_{jk} = \pm 1$, $\zeta_j = \pm 1$ and $\alpha_{rkj}$ $(k = 1, 2, \cdots, T_j; j = 0, 1, 2, \cdots, m; r = 1, 2, \cdots, n)$ are real numbers. It is a constrained GP problem with degree of difficulty (DD) $= \sum_{j=0}^{m} T_j - (n+1)$. The pseudo dual problem of the problem (4.1) is

$$\text{Maximize } d(w;\lambda) = \zeta_0 \left( \prod_{j=1}^{m} \left(\frac{c_{0k}}{w_{0k}}\right)^{\sigma_{0k}c_{0k}} \prod_{k=1}^{p} \left(\frac{c_{jk}}{w_{jk}}\right)^{\sigma_{jk}c_{jk}} \sum_{j=1}^{m} \lambda_j^{\lambda_j \zeta_j} \right)^{\zeta_0}$$

Subject to

$$\begin{cases} \sum_{k=1}^{T_j} \sigma_{jk} w_{jk} = \lambda_{jk}\zeta_{jk} & \text{for } j = 0,1,2,\cdots,m, \\ \sum_{k=1}^{p} \sigma_{jk}\alpha_{jk} w_{jk} = 0 & \text{for } r = 1,2,\cdots,n;\ j = 1,2,\cdots,m, \\ w_{jk} > 0 & \text{for } j = 1,2,\cdots,m;\ k = 1,2,\cdots,p \\ \lambda_0 = 1 \text{ and } \lambda_k \geq 0 & \text{for } k = 1,2,\cdots,p. \end{cases}$$

(It is assumed that the sign of the objective, $\zeta_0$ is known). In practical problem, it is possible to soften the rigid requirements of the DM to strictly minimize the objective function and strictly satisfy the constraints. In such situation, the concept of FGP is arise as follows

4.1. **Fuzzy Geometric Programming.** The GP problem (4.1) may be taken as following FGP problem

$$\tilde{Min}\, f_0(x) = \sum_{k=1}^{T_0} c_{0k} \prod_{r=1}^{n} x_r^{\alpha_{0kr}} \lessapprox f_0 \tag{4.2}$$

Subject to

$$\begin{cases} f_j(x) = \sum_{k=1}^{T_j} c_{jk} \prod_{r=1}^{n} x_r^{\alpha_{jkr}} \lessapprox f_j^0 & \text{for } j = 1,2,\cdots,m, \\ x_r > 0 & \text{for } r = 1,2,\cdots,n. \end{cases}$$

The symbol $\tilde{Min}$ denotes a relaxed or fuzzy version of 'Min'. It implies that the objective function should be minimized as well as possible near $g_0$. Similarly the symbol "$\lessapprox$" denotes a fuzzy version of "$\leq$". It also implies that the constraints should be well satisfied. These fuzzy requirements may be quantified by eliciting membership functions $\mu_j(f_j(x))$ $(j = 0,1,2,\cdots,m)$ from the DM for all functions $g_j(x)$ $(j = 0,1,2,\cdots,m)$. By taking account of the rate of increased membership satisfaction, the DM must determine the subjective membership function $\mu_j(f_j(x))$. It is in general a strictly monotone decreasing linear or non-linear function with respect to $g_j(x)$ $(j = 0,1,2,\cdots,m)$. Here for simplicity, linear membership functions are taken as follows

$$\mu_j(f_j(x)) = \begin{cases} 1 & \text{if } f_j(x) \leq f_j^0, \\ \frac{f_j' - f_j(x)}{f_j' - f_j^0} & \text{if } f_j^0 \leq f_j(x) \leq f_j' \text{ for } j = 0,1,2,\cdots,m, \\ 0 & \text{if } f_j(x) \geq f_j'. \end{cases} \tag{4.3}$$

Its rough sketch is given in Figure 1. According to the Figure 1,
$f_j^0$ = the value of $f_j(x)$ such that the grade of membership function $\mu_j(f_j(x))$ is 1.

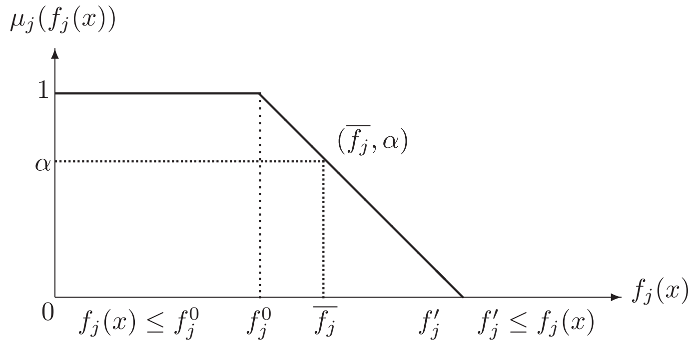


FIGURE 1. Linear membership function for $f_i \precsim f_i^0$

$f_j'$ = the value of $f_j(x)$ such that the grade of membership function $\mu_j(f_j(x))$ is 0. $\overline{f_j}$ = the intermediate value of $f_j(x)$ such that $\overline{f_j} \in (f_j^0, f_j')$ the grade of membership function say $\alpha \in (0,1)$. According to the fuzzy decision making process [3] on fuzzy objective and constraint goals using membership functions 4.3 the problem of finding the maximizing decision to choose optimal $x^*$ using operators is presented as follows

(i) FGP based on Max-Min operator.

Find $x^* = (x_1^*, x_2,^*, \cdots, x_n^*)^T$, where

$$\mu_D(x^*) = \max_{x_r>0; r=1,2,\cdots,n} \left( \min_{j=0,1,2,\cdots,m} \mu_j(f_j(x)) \right). \tag{4.4}$$

Considering membership functions (4.3), the problem (4.2) reduces to

$$Max\, V_m(x) = \frac{f_0' - \sum\limits_{i=1}^{T_0} c_{0i} \prod\limits_{r=1}^{n} x_r^{a_{0ir}}}{f_0' - f_0^0}$$

subject to

$$\begin{cases} \dfrac{f_j' - \sum\limits_{i=1}^{T_j} c_{ji} \prod\limits_{r=1}^{n} x_r^{a_{jir}}}{f_0' - f_j^0} \geq \dfrac{f_0' - \sum\limits_{i=1}^{T_0} c_{0i} \prod\limits_{r=1}^{n} x_r^{a_{0ir}}}{f_0' - f_0^0} & \text{for } j = 1,2,\cdots,m, \\ x_r > 0 & \text{for } r = 1,2,\cdots,n. \end{cases}$$

so optimal decision variables vector $x^* = (x_1^*, x_2^*, \cdots, x_n^*)^T$ with optimal objective value is

$$V_m^*(x^*) = \frac{f_0'}{f_0' - f_0^0} - V_m'(x^*)$$

where $x^* = (x_1^*, x_2^*, \cdots, x_n^*)^T$ is optimal decision variable vector of the following GP problem

$$Min\, V_m'(x) = \frac{1}{f_0' - f_0^0} \sum_{i=1}^{T_0} c_{0i} \prod_{r=1}^{n} x_r^{a_{0ir}} \tag{4.5}$$

subject to

$$\begin{cases} \frac{f_0'-f_0^0}{f_0'f_j'-f_j'f_0'}\sum_{i=1}^{T_j} c_{ji}\prod_{r=1}^{n} x_r^{a_{jir}} - \frac{f_0'-f_0^0}{f_0'f_j'-f_j'f_0'}\sum_{i=1}^{T_0} c_{0i}\prod_{r=1}^{n} x_r^{a_{0ir}} \le 1 & \text{for } j=1,2,\cdots,m, \\ x_r > 0 & \text{for } r=1,2,\cdots,n. \end{cases}$$

Now problem (4.5) is a synomial GP problem with $DD = mT_0 + \sum_{j=0}^{m} T_j - (n+1)$ and can be solved by GP technique.

(ii) FGP based on Max-Additive Operator

Find $x^* = (x_1^*, x_2^*, \cdots, x_n^*)^T$ where

$$\mu_D(x^*) = \max_{x_r>0;r=1,2,\cdots,n}\left(\sum_{j=0}^{m}\mu_j(f_j(x))\right). \tag{4.6}$$

Considering membership functions (4.3), the problem (4.2) reduces to

$$Max\, V_a(x) = \sum_{j=0}^{m}\frac{f_j' - \sum_{i=1}^{T_j} c_{ji}\prod_{r=1}^{n} x_r^{a_{jir}}}{f_j'-f_j^0}$$

subject to $x_r > 0$ for $r = 1, 2, \cdots, n$.

So optimal decision variable vector $x^* = (x_1^*, x_2^*, \cdots, x_n^*)^T$ with optimal objective value is

$$V_a^*(x^*) = \sum_{j=0}^{m}\frac{f_j'}{f_j'-f_j^0} - V_a'(x^*)$$

where $x^* = (x_1^*, x_2^*, \cdots, x_n^*)^T$ is optimal decision variable vector of the unconstrained GP problem

$$Min\, V_a'(x) = \sum_{j=0}^{m}\frac{1}{f_j'-f_j^0}\sum_{i=1}^{T_j} c_{ji}\prod_{r=1}^{n} x_r^{a_{jir}} \tag{4.7}$$

subject to $x_r > 0$ for $r = 1, 2, \cdots, n$.

Now problem (4.7) is an unconstraint posynomial GP problem with $DD = \sum_{j=0}^{m} T_j - (n+1)$ and can be solved by GP technique.

## 5. Fuzzy Geometric Programming on Reliability Optimization Problem

In the fuzzy reliability problem (2.2) linear membership functions for fuzzy objective and constraint goals are taken. The linear membership function for system reliability is

$$\mu_{R_S}(R_S(R_1,R_2,\cdots,R_n)) = \begin{cases} 1 & \text{if } R_S(R_1,R_2,\cdots,R_n) \le R, \\ \frac{(R+P_R)-R_S(R_1,R_2,\cdots,R_n)}{P_R} & \text{if } R \le R_S(R_1,R_2,\cdots,R_n) \le R+P_R, \\ 0 & \text{if } R_S(R_1,R_2,\cdots,R_n) \ge R+P_R. \end{cases} \tag{5.1}$$

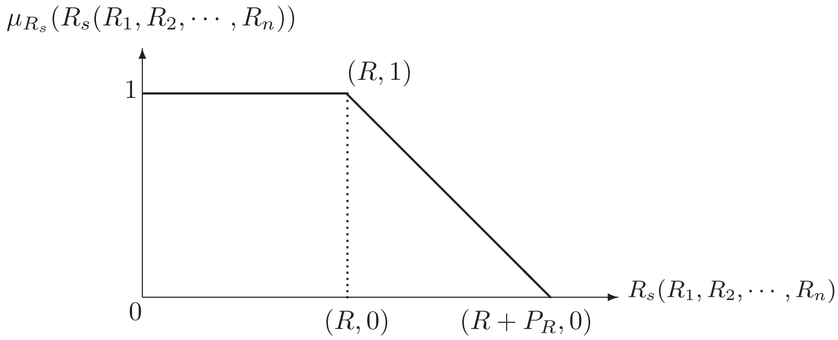


Figure 2. Membership of reliability function $R_s(R_1, R_2, \cdots, R_n) \lessapprox R$ with maximum tolerance $P_R$

Its rough sketch is given in Figure 2.

And the linear membership function for cost constraint is

$$\mu_{C_S}\left(C_S\left(R_1, R_2, \cdots, R_n\right)\right)$$
$$= \begin{cases} 1 & \text{if } C_S\left(R_1, R_2, \cdots, R_n\right) \leq C, \\ \frac{(C+P_C)-C_S(R_1,R_2,\cdots,R_n)}{P_R} & \text{if } C \leq C_S\left(R_1, R_2, \cdots, R_n\right) \leq C + P_C, \\ 0 & \text{if } C_S\left(R_1, R_2, \cdots, R_n\right) \geq C + P_C. \end{cases} \tag{5.2}$$

Its rough sketch is given in Figure 3.

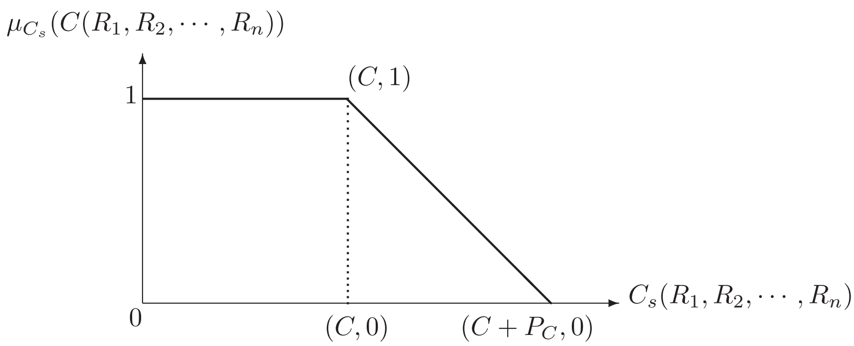


Figure 3. Membership of cost function $C_s(R_1, R_2, \cdots, R_n)) \lessapprox C$ with maximum tolerance $P_C$

Now fuzzy series system reliability model (2.2) is presented based on different operators.

5.1. **FGP on series system model (2.2) through Max-Min Operator.** Using membership function (5.1) and (5.2) through max-min operator, fuzzy reliability model (2.2) reduces to

$$Max\, \mu_{R_S}\left(R_S\left(R_1, R_2, \cdots, R_n\right)\right)$$

subject to

$$\mu_{R_S}\left(R_S\left(R_1, R_2, \cdots, R_n\right)\right) \leq \mu_{C_S}\left(C_S\left(R_1, R_2, \cdots, R_n\right)\right)$$

$0 < R_i \leq 1$ for $i = 1, 2, \cdots, n$, i.e.,

$$Max\, V_R\left(R_1, R_2, \cdots, R_n\right) = \tfrac{R+P_R}{P_R} - \tfrac{R_1^{-1}R_2^{-1}\cdots R_n^{-1}}{P_R} \tag{5.3}$$

subject to

$$\tfrac{P_R}{CP_R-RP_C}\sum_{i=1}^{n} C_i R_i^{a_i} - \tfrac{P_C}{CP_R-RP_C} R_1^{-1}R_2^{-1}\cdots R_n^{-1} \leq 1$$
$$0 < R_i \leq 1 \text{ for } i = 1, 2, \cdots, n.$$

The maximization problem (5.3) can be written as a synomial GP problem as follows

$$Min\, V_R'\left(R_1, R_2, \cdots, R_n\right) = \tfrac{1}{P_R} R_1^{-1}R_2^{-1}\cdots R_n^{-1} \tag{5.4}$$

subject to

$$\tfrac{P_R}{CP_R-RP_C}\sum_{i=1}^{n} C_i R_i^{a_i} - \tfrac{P_C}{CP_R-RP_C} R_1^{-1}R_2^{-1}\cdots R_n^{-1} \leq 1$$
$$0 < R_i \leq 1 \text{ for } i = 1, 2, \cdots, n.$$

It is synomial GP with $DD = n + 2 - (n + 1) = 1$. So, $V_R^*\left(R_1^*, R_2^*, \cdots, R_n^*\right) = \frac{R+P_R}{P_R} - V_R'\left(R_1^*, R_2^*, \cdots, R_n^*\right).$

The dual problem of primal problem (5.4) is follows

$$Max\, d_R(W) = \left(\tfrac{1}{P_R w_{01}}\right)^{w_{01}} \left(\prod_{i=1}^{n}\left(\tfrac{\lambda P_R C_i}{(CP_R-RP_C)w_{1i}}\right)^{w_{1i}}\right)\left(\tfrac{\lambda P_C}{(CP_R-RP_C)w_{1n+1}}\right)^{-w_{1n+1}}$$

subject to

$$\begin{cases} w_{01} = 1 \\ -w_{01} + a_i w_{1i} + w_{1n+1} = 0 \quad \text{for} \quad i = 1, 2, \cdots, n \end{cases}$$

where $w_{01}, w_{1i} \geq 0$ for $i = 1, 2, \cdots, n+1$ and $\lambda = \sum_{i=1}^{n} w_{1i} - w_{1n+1}$. It can be solved by usual technique.

5.2. **FGP on series system model (2.2) through Max-Additive Operator.** Based on max-additive operator using membership function (5.1) and (5.2), fuzzy reliability model (2.2) reduces to

$$Max\, \left\{\mu_{R_S}\left(R_S\left(R_1, R_2, \cdots, R_n\right)\right) + \mu_{C_S}\left(C_S\left(R_1, R_2, \cdots, R_n\right)\right)\right\}$$
subject to $0 < R_i \leq 1$ for $i = 1, 2, \cdots, n$, i.e.,

$$Max\, V_a\left(R_1, R_2, \cdots, R_n\right) = \tfrac{RP_C-CP_R}{P_R P_C} - \tfrac{R_1^{-1}R_2^{-1}\cdots R_n^{-1}}{P_R} - \tfrac{1}{P_C}\sum_{i=1}^{n} C_i R_i^{a_i} \tag{5.5}$$

$R_i > 0$ for $i = 1, 2, \cdots, n.$

| Operator | $R_1^*$ | $R_2^*$ | $R_3^*$ | $R_S^*\left(R_1^*, R_2^*, R_3^*\right)$ | $C_S^*\left(R_1^*, R_2^*, R_3^*\right)$ |
|---|---|---|---|---|---|
| Max-min | 0.929351 | 0.885740 | 0.838395 | 0.690136 | 374.467 |
| Max-additive | 0.951678 | 0.907722 | 0.859671 | 0.742634 | 383.009 |

Table 1. Optimal solutions of fuzzy reliability model (2.2) through FGP

The maximization problem (5.5) can be written as unconstrained posynomial GP problem as follows

$$Min\, V_a'\left(R_1, R_2, \cdots, R_n\right) = \frac{R_1^{-1}R_2^{-1}\cdots R_n^{-1}}{P_R} + \frac{1}{P_C}\sum_{i=1}^{n} C_i R_i^{a_i} \tag{5.6}$$

where $0 < R_1 \leq 1$ for $i = 1, 2, \cdots, n$.

It is a posynomial GP problem with $DD = n + 1 - (n + 1) = 0$. so,

$$V_a^*\left(R_1^*, R_2^*, \cdots, R_n^*\right) = \frac{RP_C - CP_R}{P_R P_C} - V_a'\left(R_1^*, R_2^*, \cdots, R_n^*\right).$$

The dual problem of (5.6) is as follows

$$Max\, d_a(W) = \left(\prod_{i=1}^{n}\left(\frac{C_i}{P_C w_{0i}}\right)^{w_{0i}}\right) \times \left(\frac{1}{P_R w_{0n+1}}\right)^{w_{0n+1}}$$

subject to

$$\begin{cases} \sum_{i=1}^{n} w_{0i} + w_{0n+1} = 1 & \\ a_i w_{0i} - w_{0n+1} = 0 & \text{for } i = 1, 2, \cdots, n, \\ w_{0i} > 0 & \text{for } i = 1, 2, \cdots, n+1. \end{cases}$$

It can be solved by conventional technique.

## 6. Numerical Example of Series System Reliability Model

In this section, we have considered the reliability problem of series system reliability model for the numerical exposure. Suppose a device has three components in series. Here the cost coefficients and indices are taken as $C_1 = 130$, $C = 140$, $C = 150$, $a_1 = 0.95$, $a_2 = 0.92$, $a_3 = 0.9$ and fuzzy objective goal i.e., system reliability is $R = 0.855$ with maximum tolerance $P_R = 0.4$, and cost constraint goal i.e., total available system cost is $C = 350$ with maximum tolerance $P_C = 35$. The optimal solutions of the fuzzy model through FGP with two operators namely maxmin and max-additive operators are presented in Table 1.

It is to be noted that according to the cost of the system and reliability measures of the series system, the reliability of the system are 0.690136 and 0.742634 respectively through the max-min and max-additive operator. Table-1 shows that optimal objective value is better through max-additive operator for this series system reliability model. However it is not mandatory that max-additive gives always better result but here we try to draw attention about FGP that we generally use max-min operator but for some cases max-additive operator may be more suitable.

## 7. Conclusions

FGP method is discussed and illustrated based on fuzzy decision-making process with application on the series system reliability model. In this paper we have taken a series system whose reliability is considered in fuzzy environment. This fuzzy reliability problem of series system model is discussed through FGP using max-min and maxadditive operator. We want to draw attention of researcher about FGP technique, since usually researcher used FGP technique based on max-min operator but some times other operator may be more suitable and gives better result, however it is depends on the problem. Several reliability and optimization problems in fuzzy environment may be solved easily for decision-making through FGP.

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